Intermittency Approach in the Nuclear Collisions of $^{28}$Si + AgBr at 14.6A GeV - Nuclear Emulsion Experiment a Particle Detector

Mohammad Ayaz Ahmad
Physics Department, Faculty of Science, P.O. Box 741, University of Tabuk, Saudi Arabia

Abstract

A new method has been applied for the study of intermittency in the collisions of $^{28}$Si + AgBr at 14.6 A GeV for the produced relativistic charged particles by using the method of mixed factorial moment (MFM). In this paper an analysis has been made for the presence of fractal behavior with three different approaches of Scaled factorial Moments (SFMs) named as- horizontal, vertical and mixed (horizontal and vertical together). The non-statistical fluctuations of relativistic charged particles have been calculated on events with different degree of centrality. These results have been compared with the results of simulated data obtained from UrQMD model and we find a good agreement between experimental data and simulated data.

Keywords: multiplicity fluctuation;, UrQMD simulation technique; scaled factorial moemnts component; charged particle production

Introduction

A phase transition from hadron gas to quark gluon-plasma (QGP) is expected to occur at high energy densities ($\approx 1$ GeV/ fm$^3$). There are some signals at the maximum SPS energies that quark-gluon-plasma is created at the early stage of heavy ion collisions [1-4]. Various theoretical models predict an increase of multiplicity fluctuations near the onset of deconfinement state [5, 6]. An approach of ‘intermittency’ phenomenon [7 and references therein] belongs to scaled factorial moment (SFM) and its scaling nature with diminishing phase-space interval size ( $\delta X$), which is mainly an arrangement with the short-range correlations among the produced particles in heavy ion collisions at high-energies. Analysis of SFM is also capable of suppressing any Poisson-type statistical noise present in the local density fluctuations and of characterizing the non-statistical component of the same in terms of a finite set of parameters. The experimental data on particle fluctuations in small space domains has been presented for different collisions at different energies [4, 6-8]. We are studying here the intermittent behaviour of multiparticle production can be related to the formation of mini-jets, a second order phase transition from quark gluon plasma to the normal hadronic matter or the random cascading process [6-9].

It has been found that the intermittency is a general property of multiparticle production in relativistic heavy ion collisions [6-9]. The intermittent behavior of multiparticle production can be related to the formation of mini-jets, a second order phase transition from quark gluon plasma to the normal hadronic matter or the random cascading process [9]. In this paper we draw some results of the analysis of the multiplicity fluctuations in pseudo-rapidity scale for $^{28}$Si (projectile) + AgBr (target) collisions at 14.6 A GeV. It has been proposed to study the dependence of scaled factorial moments $F_q$, on the order of the moment, $q$, as a function of the bin width $\Delta \eta$ [6-9], the pseudo-rapidity interval is $\Delta \eta$. The intermittent behavior should a clue of a power law dependence of the scaled factorial moments on the bin size such as follow:

$$F_q = \left( \frac{\Delta n}{\Delta \eta} \right)^{\alpha_q}, \text{ where } \alpha_q > 0$$

I. Data collections and experimental details

In the present experiment, FUJI nuclear emulsion pellicles were irradiated horizontally with a beam of $^{28}$Si nuclei at 14.6A GeV at Alternating Gradient Synchrophasotron (AGS) of Brookhaven National Laboratory (BNL), NewYork, USA. The method of line scanning has been adopted to scan the stacks, which was carried out carefully using Japan made NIKON (LABOPHOT and Tc-BIOPHOT) high-resolution microscopes with 8 cm movable stage using 40X objectives and 10X eyepieces by two independent observers, so that the bias in the detection, counting and measurements can be minimized.
The interactions due to beam tracks making an angle < 2° to the mean direction and lying in emulsion at depths > 35 μm from either surface of the pellicles were included in the final statistics. The other relevant details about the present experiments and target identifications may be seen in our earlier publications [10-17]. In the measured interactions all charged secondary particles were classified according to the commonly accepted emulsion experiment terminology into following groups:

A. **Black track producing particles** (N_b)

The tracks with specific ionization $g^* > 10$ ($g^* = g/g_0$, where $g_0$ is the Plateau ionization of a relativistic singly charged particle and $g$ is the ionization of the charged secondary) have been taken as black tracks. These correspond to protons of relative velocity $\beta < 0.3$ and range in emulsion $L < 3.0$ mm [11].

B. **Grey track producing particles** (N_g)

The tracks with specific ionization $1.4 \leq g^* \leq 10$ corresponding to protons with velocity in the interval $0.3 \leq \beta \leq 0.7$ and range $L \geq 3.0$ mm in nuclear emulsion are called grey tracks [11].

C. **Shower track producing particles** (N_S)

The tracks with specific ionization $g^* < 1.4$ corresponding to protons with relative velocity $\beta > 0.7$ are classified as shower tracks. These tracks are mostly due to relativistic pions with small admixture of charged K-mesons and fast protons. In order to eliminate all the possible backgrounds due to $\gamma$ overlap (where a $\gamma$ from a $\pi^0$ decay converts into $e^+ e^-$ pair) close to shower tracks near vertex, special care was taken to exclude such $e^+ e^-$ pairs from the primary shower tracks while performing angular measurements. Usually all shower tracks in the forward direction were followed more than 100-200 μm from the interaction vertex for angular measurement. The tracks due to $e^+ e^-$ pair can be easily recognized from the grain density measurement, which is initially much larger than the grain density of a single charged pions or proton track. It may also be mentioned that the tracks of an electron and positron when followed downstream in nuclear emulsion showed considerable amount of Coulomb scattering as compared to the energetic charged pions. Such $e^+ e^-$ pairs were eliminated from the data.

D. **Projectile fragments** (Z)

The projectile spectator fragments, singly and multiple charged, emitted inside fragmentation cone, usually we determine the number of alpha particles ($n_\alpha$) and the number of fragments with $Z > 2$, separately [9].

The polar ($\theta$) and azimuthal ($\phi$) emission angles of all tracks have been measured, and the pseudorapidity has been calculated by $\eta = -\ln \tan(\theta/2)$ for each shower particle.

II. **Mathematical analysis**

We used three methods of analysis; method of horizontal factorial moments (HFM), vertical factorial moments (VFM) and published in [6-8] and also in the present analysis we used the mixed factorial moments (MFM). The standard horizontal factorial moments $F^{(H)}_e$ characterizing the $e$th event are defined by the following relation:

$$F^{(H)}_e(q) = M^{q-1} \sum_{m=1}^{M} \frac{F(n_{me}, q)}{[\eta^q(n_{me}, q)]},$$

(2)

where $M$ is the number of equal bins of size $\delta \eta$ into which the pseudorapidity interval $\Delta \eta$ has been divided, $n_{me}$ is the number of shower particles in the $m^{th}$ bin. Non averaging and non normalized factorial moments are given by:

$$F(n_{me}, q) = n_{me} (n_{me} - 1) \cdots (n_{me} - q + 1)$$

(3)

The vertical averaging of $F^{(H)}_e$ gives the full form

$$F^{(H)}(q) = \frac{1}{E} \sum_{e=1}^{E} F^{(H)}_e(q),$$

(4)
where \( E \) is the total number of events. The denominator of the horizontal moment (Eqn. 2) is 
\[
N_e^{(H)} = \sum_{m=1}^{M} n_{me}.
\] 
(5)

The vertical analysis is suggested in case of rare events with sharp peaks [8, 9]. The normalized standard vertical moments characterizing the \( m^{th} \) bin are given by
\[
F_m^{(V)}(q) = E^{-q-1} \sum_{e=1}^{E} \frac{F(n_{me}, q)}{[N_m^{(V)}]^q}
\] 
(6)

and the horizontal averaging gives the full form
\[
F^{(V)}(q) = \frac{1}{M} \sum_{m=1}^{M} F_m^{(V)}(q),
\] 
(7)

where; \( N_m^{(V)} = \sum_{e=1}^{E} n_{me} \) is the sum of multiplicities which appear in the \( m^{th} \) bin of all events. Besides the horizontal and vertical factorial moment methods a mixed approach is applied. The mixed factorial moments are defined such as:
\[
F^{(HV)} = M^{-q-1} E^{-q-1} \sum_{m=1}^{M} \sum_{e=1}^{E} \frac{F(n_{me}, q)}{[N^{(HV)}]^q}
\] 
(8)

where; \( N^{(HV)} = \sum_{m=1}^{M} \sum_{e=1}^{E} n_{me} \) is the total number of charged particles observed in the sample of \( E \) events.

III. results and discussions

Figure 1, shows the normalized pseudorapidity distributions (i.e. the particle number densities in rapidity space) of the secondary charged shower particles emitted in the interactions of \( ^{28}\text{Si-Em} \) at 14.6A GeV. In the same figure the results obtained from the interactions of \( ^{28}\text{Si-Em} \) and \( ^{12}\text{C-Em} \) at 4.5A GeV along with the result of \( ^{16}\text{O-AgBr} \) at 3.7, 60 and 200A GeV respectively have been shown for comparison.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** The pseudorapidity distributions of the shower charged particles produced at various energies

One may notice that the \( \eta \)-distributions are completely scaled in the region of smaller values of \( \eta \) and also found to be independent of mass of incident beam whereas weak energy dependence has been found in this region. The distribution
is broader for higher mass as well as beam energy. The height of the centroid increases many times in the case of nucleus-nucleus collisions with respect to the proton-nucleus collisions [6-9].

It is well known that the statistical fluctuations are suppressed in the study of scaled factorial moments (SFMs), whereas, ordinary multiplicity moments \( < n^q > / < n >^q \) is unable to reveal the existence of dynamical fluctuation due to significant contribution of the purely statistical fluctuations. The SFMs are capable of measuring the large-scale fluctuations and provide information about the pattern of these fluctuations. The cumulative interval of \( X(\eta) \) or \( X(\phi) \) variable, is successively divided into \( M = 2-30 \) bins. Recently, it has been suggested to study the interaction dynamics and sources of the multiplicity fluctuations and power-law scaling behaviour of SFMs in the multi-dimensional \( (\eta, \phi, \ldots) \) phase-space. Thus analysis of the data has been performed to check the existence of intermittency signal from one-dimensional \( \eta \) or \( \phi \) variable to the two-dimensional \( (\eta, \phi) \) variable. One started with a rectangle in \( (\eta, \phi) \)-space. The rectangle was divided into \( M_{\eta,\phi} \) bins each of size \( (\delta\eta, \delta\phi) = (\Delta\eta/M_{\eta}, \Delta\phi/M_{\phi}) \). Fig. 2 (a-c) shows the plots of \( \ln < F_q > \) as a function of \( \ln M_{\eta} M_{\phi} \) in the interactions of \(^{28}\)Si-Em collisions at 14.6A GeV for \( q = 2-6 \) for different \( N_S \)-intervals. A linear rise of SFMs is seen from the figure and a larger value of \( \alpha_q \) is reported in Table I. A stronger intermittency effect is observed in 2D, in comparison with a weak signal observed in one-dimension (1D). Result obtained using UrQMD model is also shown in Fig. 2 (a-c) for comparison, which also follows the similar trend. The individual analysis in pseudorapidity or azimuthal space may not be favourable to detect the existence of quark-gluon plasma. Instead of QGP, intermittent behaviour is observed. The analysis of experimental result in two-dimensions is more effective than the one-dimension in different \( N_S \)-intervals. Finally, it may be concluded that the results in nucleus-nucleus interactions with \(^{28}\)Si beam at 14.6A GeV fulfil the predictions of the self-similar cascade model in observing the intermittency.

### TABLE I. THE VALUES OF INTERMITTENCY INDICES IN TWO DIMENSIONAL (2D) PHASE SPACES FOR NUCLEAR COLLISIONS AT AN ENERGY 14.6 A GEV/C

<table>
<thead>
<tr>
<th>Order of Index</th>
<th>Values of Intermittency Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 8 \leq N_S \leq 15 )</td>
<td>( 8 \leq N_S \leq 15 ) ( N_S \geq 24 )</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>0.175±0.010</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>0.395±0.011</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td>0.689±0.014</td>
</tr>
<tr>
<td>( \alpha_5 )</td>
<td>1.183±0.018</td>
</tr>
<tr>
<td>( \alpha_6 )</td>
<td>1.767±0.025</td>
</tr>
</tbody>
</table>

![Graph showing intermittency indices](image-url)
Figure 2. The Variations of $\ln<F_{q}(\eta\phi)>$ as a function of $\ln M_{\eta\phi}$ in $\eta\phi$-phase space for different $N_S$ -intervals in the interactions of $^{28}$Si-Em with UrQMD and MC-RAND model

The error bars in Figs. 2 (a-c) are estimated by considering them as independent statistical errors only and the solid lines drawn indicate the least squares fit to the respective experimental data points. Though the effect of point-to-point correlations of the statistical errors for different bin sizes has not been taken into consideration in the present study, it is expected that the exclusion of the correlation of the statistical errors will not change the main result appreciably. For better understanding of intermittency indices, $\alpha_q$, it has been calculated using least square fitting of the data points in Figs. 2 (a-c). The values obtained experimentally in different $N_S$ -intervals are reported in Table I in $\eta\phi$ -phase (2D) space respectively. It is observed from the table that the exponent, $\alpha_q$, in Eqn. (1) increases with increasing order of moments, q, for different sets of data samples in 2-D phase space.

The values of $\alpha_q$ in are consistently higher for multiplicity interval $N_S \geq 2l$ than rest of another two intervals values in $\eta\phi$ -space (2D). It is also observed that the intermittency effect is not independent of the phase space variable. The intermittency effect is more pronounced in $\eta\phi$ -space (2D) than in one-dimension. The observation of experimental results clearly indicates a stronger intermittency signal in $\eta\phi$ -space. The higher values of $\alpha_q$ in $\eta\phi$ -space is consistent with the interpretation that the increase in factorial moments is due to clusters of particles yielding a correlation between local $\eta$ and $\phi$ -fluctuations. However, a constant and small values of $\alpha_q$ are seen for MC generated events.

It has been observed [7-9] that if the dynamics of intermittency is due to self-similar cascading, then there is a possibility of observing a non-thermal phase transition, which is believed to occur during the collision. If such a non-thermal phase transition is present then the function $\lambda_q$ should have a minimum at certain value of $q = q_c$, where $q_c$ is some minimum
point in the distribution [7-9]. The region with $q < q_c$ is dominated by numerous small fluctuations, whereas the region with $q > q_c$ is due to rarely large fluctuations. This situation can easily be compared with a mixture of “liquid” of large number of small fluctuations and a “dust” consisting of few grains of very large density.

The “liquid” and the “dust” phases coexist. The variation of $\lambda_q$ as a function of $q$ for grey and black particles produced in $^{28}$Si-Em interactions at 14.6A GeV has been shown in Fig. 3. The result of $^{16}$O-AgBr at 4.5A GeV has also been shown for comparison purpose. From the figure we observed that no clear-cut minimum value of $\lambda_q$ for certain value of $q$ has been observed within the limit $2 \leq q \leq 6$ as reported by other workers [16,17]. However, a little deviation of the experimental data from the "no intermittency" line ($\alpha_q = 0$) indicates the presence of a weak intermittency effect in $\cos \Theta$-space for the present data. A weak intermittency effect is found due to intermixing of many subprocesses during multiparticle production of cascade mechanism. Thus it may be concluded that data for grey and black particles do not support a clear evidence for the existence of non-thermal phase transition. To get more unambiguous evidence, the analysis should be done upto $q = 8$ with large statistics at high energies and with different projectiles.

IV. conclusions and final remarks

With an outlook of the present article entitled “Intermittency Analysis by Mixed Scaled Factorial Moments in the Nuclear Collisions of $^{28}$Si + AgBr at 14.6A GeV” describes the study about the dynamics of multiparticle production in nucleus-nucleus collisions. The analysis is based on 951 interactions of $^{28}$Si+AgBr collisions at 14.6A GeV using nuclear emulsion. One can conclude the followings from the present article:

The generalized power law behaviour of mixed scaled factorial moments in in two-dimension (2D), $\eta \phi$ phase space reflects an evidence for an intermittency pattern of fluctuations in all the above mentioned interactions at 14.6A GeV for shower particles. The present study also gives a strong evidence of self-similar structure in multiparticle production in such collisions at 14.6A GeV for two-dimensional $\eta \phi$-phase space. The values of the intermittency indices, $\alpha_q$, in two dimensional-space show a strong dependence on $N_S$-multiplicity as well as phase-space dimension.

Some significant results have been obtained for dynamical fluctuations in present work. It is believed that these fluctuations may be a weak signal of QGP formation in such experiment. Further, evidence of these fluctuations has also been observed in low energy nuclear collisions, whereas the formation of QGP is not expected. Even in target fragmentation process, where the QGP phase transition is most unlikely, some physicists have reported evidence of dynamical fluctuations in earlier work.

Figure 3. The Dependence of $\lambda_q$ on the order of moments, $q$ in the collisions of $^{28}$Si+AgBr nuclei at 14.6A GeV.
Acknowledgment
The author would like to acknowledge the keen support for this work of the Department of Physics, Faculty of Science, University of Tabuk, Saudi Arabia.

References